APPLICATION MULTipoLE APPROACH FOR SYNTHESIS
OF ELECTROSTATIC FIELDS

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Multipole-cylindrical field for different contributions of cylindrical field and circular multipole was considered. Equipotential portraits of electrostatic hexapole-cylindrical and decapole-cylindrical fields were obtained. The shape of the deflecting outer electrode defining the spatial distribution of field for schemes of energy analyzers with hexapole-cylindrical and decapole-cylindrical fields was determined. Schemes of mirror type energy analyzers based on electrostatic hexapole-cylindrical and decapole-cylindrical fields were proposed. Electron-optical characteristics of the proposed schemes were calculated.

Keywords: electrostatic field, circular multipole, multipole-cylindrical field, equipotential portraits, mirror type energy analyzer, hexapole, decapole.

INTRODUCTION

In recent decades, the methods of electron spectroscopy of solid surface received intensive development in physics research and monitoring of a number of modern production processes. The basis of these methods is the analysis of charged particles beams by spectrometers. Spectrometers spatially separate of energy charged particles beams and focus share allocated beams on the receiving devices. The main part of the spectrometer is an energy analyzer.

The main part of the focusing elements of electron optics is based on the axial - symmetrical electrostatic and magnetic fields. Analysis of devices, which used in the methods of electron spectroscopy (Auger spectroscopy, photoelectron spectroscopy, energy loss spectroscopy, etc.), indicates that the analytical instrumentation adopted by the orientation to the mirror axial-symmetric optics. This is due to two main reasons: high luminosity, high energy resolution and sensitivity. Mirror type cylindrical and spherical electrostatic energy analyzers received wide application [1,2]. Various combinations of cylindrical and spherical mirrors have been well studied [3,4].

Increased requirements on the level of the experiment dictate necessitate further development of analytical instrumentation. It is supported by the increased technological capabilities of manufacturing electrodes of different shapes. The question arises on further ways of development and calculation of electrostatic mirror analyzers, more efficient compared with the already known.

Class of axial-symmetric electrostatic field that allow separation of variables in the equations of motion and finding trajectories in quadrature or elementary functions, is extremely narrow and already well understood. These are the following fields and their superposition: cylindrical field \((V \sim \ln r)\) [1], spherical field \((V \sim 1/r)\) [2], hyperbolic field \((V \sim z^2/vr^5)\) [5], the superposition of these fields [3,4,6] uniform field \((V \sim E)\), directed along the symmetry axis [7]. Potential opportunities of this classical fields as energy analyzers of charged particle beams is largely exhausted, some of which have long been used in the electron spectrometers. For further development of the theory of electrostatic mirrors in focusing mode on the basis of axially-symmetric fields must take the next step - to consider the fields that do not allow determination of trajectories in quadrature.

The basic trajectory of a particle in an electrostatic axial-symmetric mirror has a vertex in the field and, in general, can not be described by an elementary function. Such a plane trajectory in [8,9] is called "return". "Return" trajectory of charged particles consists of two symmetric with respect to the top of the branches. Finding the return trajectories by previously used methods and subsequent search, do not allow separation of variables in the equations of motion and finding trajectories in quadrature or elementary functions is carried out only by numerical methods. Such an approach to solving the problem is complicated and laborious. However, we can greatly facilitate the task if approximate - analytically describe the return trajectories in an axial-symmetric electrostatic fields using optimally selected superposition of power series.
An approximate-analytical method for calculating the trajectories of charged particles in a cylindrical-multipole field (CMF) was proposed in [8]. This approach allows in the development of high-resolution mirror analyzers to determine their electron-optical characteristics. Calculation and analysis of the electron-optical characteristics of an electrostatic hexapole-cylindrical field (HCF) on the basis of superposition of cylindrical mirror and circular hexapole [10] were proposed in [11]. In [11] the HCF is considered, the potential of which is described in a coordinate system $r, Z$ the following expression:

$$U(r, Z) = \mu \ln r - U_h(r, Z),$$  \hspace{1cm} (1)

where $U_h(r, Z) = \frac{1}{2} \left[ Z^2 - \frac{r^2}{2} - \frac{1}{2} \right] \ln r + \frac{r^2}{2} - \frac{1}{2}$ - circular hexapole; $\mu$ – the coefficient defining the weight contribution of the cylindrical field $\ln r$. Electron-optical parameters of the energy analyzer with HCF (1) for $\mu = 5/2$ were calculated. More promising scheme of energy analyzer is obtained for the case of HCF with potential

$$U(r, Z) = \mu \ln r + U_h(r, Z),$$  \hspace{1cm} (2)

where $\mu=5/2$. For it trajectories of the charged particle beams and electro-optical properties, which were better than those of the above system, were calculated [12].

Interest is the following superposition of cylindrical field $U = \ln r$ and circular hexapole $U_h(r, Z)$,

$$U(r, Z) = \mu \ln r + \gamma U_h(r, Z),$$  \hspace{1cm} (3)

where $\gamma$ – weight component of circular hexapole.

Let consider a cylindrical multipole-field (3) for different coefficients $\mu$ and $\gamma$, that is, for the various contributions of the cylindrical field and circular hexapole. Review and analysis of the research studied and new cases of superpositions of these fields, that is, different combinations of coefficients $\mu$ and $\gamma$, will predict the prospect of choosing a particular scheme. Calculations of electron-optical characteristics of each scheme by numerical or approximate-analytical methods were very complicated. Therefore, a preliminary analysis will be concluded, first, whether or not for this scheme to carry out further calculations, and, secondly, in what direction to go when choosing the coefficients $\mu$ and $\gamma$. For calculations of equipotential portraits of multipole-cylindrical field used method of successive approximations (programming language Fortran), in addition to the calculations used MathCAD.

**Structure of superposition of multipole and electrostatic cylindrical field**

Let consider the motion of charged particles in the HCF (3), shown schematically in Fig.1. Field is formed in the space between the two axisymmetric coaxial electrodes, of which the inner cylinder of radius $r_0$ is at ground potential, the outer electrode with a curved profile at the potential $U_0$, like charges of the particles in the field (3). The problem is solved in cylindrical coordinates, two of which $Z$ and $r$ are independent, angular coordinate is cyclic due to the axial symmetry. Later the origin is convenient to move to the the trajectory top of the point $m$ and go to the coordinates $x$ and $\xi$ (Fig. 1). According to the previously adopted procedure all linear dimensions will be expressed as a percentage of the radius $r_0$ of the inner cylindrical electrode

$$x = \frac{X}{r_0} = \frac{r_m - r}{r_0} = \rho_m - \rho, \hspace{1cm} \frac{r}{r_0} = \frac{r_0 + \rho \rho}{r_0} = 1 + \rho, \hspace{1cm} \xi = \frac{Z}{r_0}.$$  \hspace{1cm} (4)

We are interested in the shape of the electrode 3. To determine the shape of the electrodes that define the spatial distribution of the field, should be explored equipotential portraits circular multipole. Fig.2 shows a family of equipotential cylindrical multipole having a symmetry plane. With the addition fields the central circle of multipole aligned with the zero equipotential logarithmic field.

**Hexapole- cylindrical field**

Make representation of the structure of the circular hexapole. In turn, a significant contribution to the circular hexapole of hexapole - cylindrical field was determined by calculation and subsequent received equipotential portraits. Series of Fig. 3 (a-d) are a family of equipotential of circular hexapole.

$$U_h(r, Z) = \frac{1}{2} \left[ \ln (1 + \rho) \left[ \frac{\xi^2 - (1 + \rho)^2}{2} - \frac{1}{2} \right] + \frac{(1 + \rho)^2}{2} - \frac{1}{2} \right]$$  \hspace{1cm} (5)
Fig. 1. The trajectory of charged particles in the HCF.
1- trajectory, 2 - cylindrical electrode, 3 - curved electrode.

Circular hexapole interesting to implement axial-symmetric electrostatic mirror consisting of a cylindrical electrode \( \rho = 0 \), the two front electrode with a zero potential, as well as axisymmetric deflecting electrode potential, whose forming coincides with one of the potential of the field. In the field region of the mirror beam of charged particles can enter and withdraw from it through the appropriate aperture window in the cylindrical or front electrodes. In the first series of fig.3 (a-d), the coefficient \( \gamma \), defining the contribution of a circular hexapole, has the following values: \( \gamma = 0.2; 0.5; 0.7; 0.8; 1 \). According to figures, we can observe how gradually with increasing \( \gamma \) structure of the field changes.

Equipotential portraits for different values of \( \gamma \) were obtained. In Fig.3 we can see that with an increase of the variable parameter \( \gamma \) becomes noticeable circular curve lying in the interval CreateMesh \((R, -2, 2, 0.1)\). For each type of symmetry with respect to the plane \((\xi = 0)\) represented field formed by hexapole sets of electrodes carrying an alternating potential. Data were obtained using the the MathCad. Fig. 4 shows three-dimensional view of equipotential field of circular hexapole.
Fig. 3. Equipotential of circular hexapole with variable parameters $\gamma$;

a) $\gamma = 0.2$;  
b) $\gamma = 0.5$;  
c) $\gamma = 0.7$;  
d) $\gamma = 0.8$;  
e) $\gamma = 1$

Fig. 4. Three-dimensional view of equipotential field of circular hexapole

The scheme of the energy analyzer based on HCF with the front electrodes is shown in Fig. 5, in which the proportion of cylindrical field and hexapole are, respectively, $\mu = 1$ and $\gamma = -1$. Forming of the curved deflecting electrode 3 (Fig. 1) is chosen in the place of one of the equipotential lines in Fig. 5, the inner cylindrical electrode shows a straight $r = 1$. The energy analyzer comprises an inner cylindrical electrode (1), the outer electrode (2), two front electrodes (3), the ring A source and ring B detector, the entrance and exit ring slits. HCF is formed two coaxial electrodes, the inner of which is cylindrical, the outer electrode,
which coincides with the equipotential of HCF, has a curved profile, convex with respect to the surface of the inner cylindrical electrode.

![Diagram](image)

**Fig. 5.** Scheme of the energy analyzer based on hexapole-cylindrical field with the front electrodes

One of the problems in creating real cylindrical spectrometer is the organization of the protection of the working volume of the influence of the boundary fields. In real spectrometer length of cylindrical electrodes in the direction of the symmetry axis is limited, conditions at the fronts can significantly affect the field distribution in the passage of the analyzed particles. To solve this problem, we offer a new mirror analyzer based on HCF, the longitudinal dimensions of which are limited to the front electrode. Analysis of constructed equipotentials families of superposition of cylindrical field and circular hexapole allows the following conclusions: main advantage of the proposed mirror analyzer based on HCF is the presence of two front electrodes, which are together with the internal cylindrical electrode at zero potential. Front electrodes defining a longitudinal size of analyzer and minimize the influence of boundary fields on the distribution of the braking field in the passage of the trajectory of the analyzed particles. From the calculations it is established, that the distance along the internal surface of the cylinder between the front electrodes in units of \( r_e = 2\sqrt{2} \). At small values \( \rho_m \) the surface forming of the front electrodes lying in the axial plane, close to a straight line having a slight deviation from the radial plane of the mirror. For example, in the case focusing "the ring-axis" slope angle to the symmetry axis is 87.3°, which means that surface of front electrodes can be approximated by a conical surface with a small deviation from the radial plane.

**Decapole-cylindrical field**

Consider decapole-cylindrical field (DCF), the potential of which is described as follows:

\[
U(\rho, \xi) = U_d(\rho, \xi) + \mu \ln(1 + \rho)
\]

(6)

where \( U_d(r, Z) = U_0 \left[ \rho \left( \frac{\xi^4}{5} - 2\rho^2 \xi^2 + \frac{1}{5} \rho^4 \right) \right] \).

Fig.6 shows the equipotential portraits of DCF for different contribution of cylindrical field \( \mu \). Portraits are the same values of \( \mu \), but with different values of \( \rho \). Fig. 6 shows that, when \( \mu < 0 \) (series b, c), 1 and 3, 8 and 10 regions, when \( \mu > 0 \) (series d, e) 6 and 7, 5 and 4 regions begin to unite.

Fig.7 shows equipotential portraits of DCF for various contributions \( \mu \) and \( \gamma \). When \( \gamma = -0.1 \), \( \mu = 2.5 \) for series a and b 1 and 3, 8 and 10 regions begin to unite, the combined areas are located far from each other. On c and d series the same field is combining and places mutually close. Fig.8 shows a three-dimensional view of a equipotential field circular decapole. Fig.9 shows the electron-optical scheme of the mirror energy analyzer with DCF (6), in which the proportion of cylindrical field and decapole respectively \( \mu=1 \) and \( \gamma = -\frac{1}{100} \). An axial trajectory of particles beam exiting from an ring source A.
The electrostatic energy analyzer with DCF comprises an inner cylindrical electrode (1), the outer electrode (2) having a curved profile, the ring A source and ring B detector, entrance \( l' \) and exit \( l'' \) slits. DCF is formed in the space between two coaxial axial-symmetric electrodes of which has an inner electrode has cylindrical shape (radius \( r_0 \)) and is at ground potential; the outer electrode having a curved profile, the deflecting potential \( U_0 \) is applied. Because of the small component of the circular field distribution of equipotentials of DCF has a slight deviation from the straight line that is close to the distribution of cylindrical field equipotentials. The analyzer operates as follows. Charged particles beam of coming from the ring source A, reflects by field of mirror and focuses in the ring image B.

Fig. 6. Equipotential portraits of decapole-cylindrical fields
Fig. 7. Equipotential portraits of decapole-cylindrical fields for different $\gamma$ and $\mu$

Fig. 8. Three-dimensional view of a circular equipotential decapole field

Fig. 9 – Scheme of the energy analyzer based on decapole-cylindrical field

Scheme of mirror decapole-cylindrical analyzer (Fig. 1) corresponds to the following electron-optical characteristics: reflection parameter binding the geometrical and energy parameters, $P = 0.55$, entry angle of axial trajectory is $\alpha = 36.8346^\circ$, the initial angular spread of the side branches is $12^\circ$, ($\Delta\alpha = \pm 6^0$). The width of the image line near the Gaussian focus caused by the initial angular divergence of the beam was determined by numerical method for calculating the trajectories and for the angle $12^\circ$ is value $\Delta l = |l(\alpha) - l(\alpha)| = 0.0076$. The width of image line of the particles beam with the same angular spread in the cylindrical mirror analyzer is about 5 times greater, $\Delta l = 0.036$.  

\[
\frac{\alpha \rho}{\mu} = -1,1
\]

\[
\frac{\alpha \rho}{\mu} = -3,3
\]
The value of specific energy dispersion characterizing resolution of decapole-cylindrical analyzer calculated for particles with an initial angular spread of 12° to equal \( \delta = \frac{D}{\Delta l} = 313.6 \), that is twice the value of the specific dispersion cylindrical mirror analyzer \( \Delta l(\pm 6) = 0.036 \), \( 5.6 \times 155.6 \times 0.036 \).

Conclusions

Multipole-cylindrical field for different contributions of cylindrical field and circular multipole was considered. Equipotential portraits of such fields were constructed and analyzed and options for their use in electron spectroscopy were discussed. Calculation, construction and analysis of equipotential portraits allowed to choose promising from a theoretical point of view variables \( \mu \) and \( \gamma \).

It is shown that the class of axial-symmetric multipole-cylindrical fields has undoubted advantages for creating schemes of energy analyzers with greater functionality. This class of electrostatic fields can be considered as very promising to solve common problems electron spectroscopy.

REFERENCES