An integral part of improving the quality of control and receiving risk assessments is the development of simulation models that predict the possible dynamics of natural disasters in high-risk regions. One of the main factors influencing the pattern of flooding is a relief. In the paper we propose a method of “successive pools” to simulate flood zone area during emergency situations using a digital elevation model. The method allows to hide the non-linearity in natural relationship between the pools and simulate the movement of large flows of water.

Keywords: floods, digital elevation models, calculation of flooding zones

Floods deal an enormous damage to the environment and present a danger for people’s life that causes relevance of fight against these elements. For many provinces of Kazakhstan the greatest danger is constituted by floods in the period of spring tide and stream-ice on rivers and also flooding of the localities, related to destruction of dams of the storage pools, washing away of protective dikes [1]. Harm which is done by floods to mankind is huge, especially if to consider not only direct, but also indirect damage.

Integral part of improvement of quality of control and receiving estimates of danger is development of the mathematical models ([2]-[3]), allowing to predict possible dynamics of development of natural disasters in regions of the increased risk. Thus one of the main factors, influencing a picture of flooding is the land relief.

This approach is based on representation of a surface of the earth in the form of the pools filled by water, the relative depth and the filled volume corresponding to the digital elevation model (DEM) to each of which is delivered in compliance (fig.1).

![Fig.1. Representation of a surface in the form of successive pools](image)

As structure of data for representation of a surface it is best of all to use structure "Knots (pools), the direction of edges and weight of edges". In this structure each knot may contain the directed edges as from it to its descendants (to knots of the following level), and to it from knots of the previous and current level. Thus, the structure represents a tree, from knots forming it (pools) (fig. 2).
The use of similar structure of data allows to increase significantly the speed of work of algorithms of analysis of this model of a surface and allows to use the mathematical tool of the theory of graphs and the theory of fractals. We will give an algorithm of calculation of zones of flooding on the basis of the considered treelike structure below. We assume that the flood plain of the river (a barrier or something like that) doesn't allow to pass the big volume of water, a difference between coming and leaving waters in a time point of \( t \) is known.

1. Construction of treelike structure

We will consider piecewise and local sites of a surface where allegedly the flooding zone can be formed. After that it is necessary to check whether contain contours of pools with boundary pools with smaller height. If it is true, it means that water will flow out for the borders of surface. It shows that the zone of flooding is found, and it is added in the list of nodal points. The treelike model of system allows to work with the mathematical tool of the theory of graphs. We will designate the initial volume of plum of water as \( V \).

2. Calculation of volumes of pools

If DEM (digital elevation model) is presented, we can calculate the volume which is necessary for filling of a certain pool. At the calculation of volume of the given knot we will apply two-dimensional interpolating in space for more exact determination of volume on the basis of approximate integrated methods. For this we will calculate additional points on the bottom of the pool by given points of DEM. Let the knots of interpolation be located in points

\[ x_j, y_k \text{ at } j \in \{1, \ldots, n\}, k \in \{1, \ldots, m\}, \]

where points by corresponding coordinates taken from DEM and meanings of heights of \( z_{jk} \) (bottom of the pool) are known in them (fig. 3).

We will construct expression of a polynom of the generalized Lagrange’s form \([4]\). Designate that:

\[ W(x) = (x - x_1) \times \ldots \times (x - x_n), \quad \bar{W}(y) = (y - y_1) \times \ldots \times (y - y_m), \]

and

\[ W_j(x) = \frac{W(x)}{x - x_j} = (x - x_1) \times \ldots \times (x - x_{j-1})(x - x_{j+1}) \times \ldots \times (x - x_n), \]
\[ \bar{W}_k(y) = \frac{\bar{W}(y)}{y - y_k} = (y - y_1) \times \ldots \times (y - y_{k-1})(y - y_{k+1}) \times \ldots \times (y - y_m). \]
then

$$f(x, y) = \sum_{j=1}^{n} \sum_{k=1}^{m} z_{jk} W_j(x) W_k(y).$$  

(1)

Further, using approximate methods of calculation of integrals and formula (1), we calculate volumes of all considered knots with the help of known methods.

![Fig. 3. Digital elevation model of a surface](image)

### 3. The calculation of coefficients of speed of filling of contiguous knots

The coefficient of the plum from one knot to another is calculated on the surface area of border of contiguous knot with knot of the top level (fig. 1), thus we will equate total coefficient of border of plum of knot of the top level to 1. In the considered model the most problem is to calculate the direction of edges. Thus the direction from the very first knot to contiguous knots is always known whereas the direction between the following contiguous knots needs to be calculated by the method mentioned below (fig. 2).

Firstly, we will consider that the amount of water of V from above is unlimited, assuming that water will spill to all pools. This assumption allows to learn the direction of edges between single-level knots.

Further the coefficients of speed of filling of contiguous knots are calculated. For the first pool the coefficient is calculated easily on the area of border with contiguous knots of the lower level (fig. 2). Thus, the volume of water flowing from the knot, and also the speed of filling of pools of contiguous knots will be known. Considering popularity of coefficients of the first knot, we will use the consecutive counter of filling of knots. As soon as the pool is filled, from the current knot we will take the sum of leaving coefficients which are equal to 1. We will assume (fig. 2) that number 3 pool were filled first. Then from the 3rd knot the directed edges will leave for contiguous knots. If we use the consecutive counter of filling of pools, then each knot (except very first) will have entrance edges with the calculated coefficients (we will know, what volume of water will come to this knot and how many will remain in the pool of this knot by them). At once after the filling of the pool it is possible to expose the directed edges to contiguous knots.
In our case the set of laws and rules by which water moves, is set by a land relief from the digital card and laws of hydrodynamics of the water flows moving on a surfaces depending on hydrodynamic laws. These laws of "protection" are in a set of laws and rules by which the structure of the flooded pools develops.

The change of conditions in contiguous pools happens simultaneously and in parallel, but time goes discretely. Despite the seeming simplicity of movement of a stream, natures of mutual relationship between pools can be various and complicated. And here in a role can enter rules of cellular automats. Cellular automats can apply on a role of the universal tool, which allows to analyze and model the most difficult behavior of nonlinear dynamic systems and allow to describe the mechanisms of processes which cannot be described by other methods. In other words, cellular automats are a technique of representation of a task which sets before itself the purpose splitting a big task into a set of discrete, small tasks in such a way that the task formulation for one element simultaneously is the formulation of all task for all elements.

As soon as at all contiguous knots all directed edges will be found (fig. 4), knowing volumes of all considered pools and total volume of the spilling water, it is possible to define the direction of a stream and the map of filling of pools.

Fig. 4. Defined direction of flooding water

For scoping of the water flowing from the river it is necessary to know the speed of a watercourse and the area of live river section. Using well-known Shezi’s formula for calculation of average speed of streams for stabilized constant motion of liquid in the region of quadratic resistance in case of stream without any pressure it gives value of rivers speed as an outcome:

$$v_{av} = C \sqrt{R \cdot I},$$  \hspace{1cm} (2)

where $v_{av}$— average speed of a stream, meter/second; $C$ — coefficient of resistance of friction on length (Shezi’s coefficient), being the integrated characteristic of forces of resistance; $R$ — hydraulic radius, m; $I$ — hydraulic slope, m/m.

The coefficient of resistance of $C$ can be calculated through the coefficient of losses on the friction of $\lambda$ by the following formula:

$$C = \sqrt{\frac{\beta \varrho}{\lambda}},$$  \hspace{1cm} (3)

And also the coefficient of resistance from a formula (3) can be determined by N. N. Pavlovsky's formula:
where $n$ — coefficient of roughness, characterizing a condition of a surface of the riverbed [6]; $y$ — index of degree, depending on the size of coefficient of a roughness and hydraulic radius:

$$y = 2.5\sqrt{n} - 0.13 - 0.75\sqrt{R(\sqrt{n} - 0.1)}$$  

(5)

The formula (5) is suitable for the vast majority of the rivers proceeding on the territory of Kazakhstan. At a value $y = 1/6$ the Shezi’s formula (2) is brought to the Manning’s formula [7].

There are other empirical formulas for determination of coefficient of resistance by C. For wide enough riverbeds of the rivers the hydraulic radius of $R$ is accepted equal to depth of a stream of $d$, i.e. $R=d$ and then the Shezi’s formula (2) for determination of average speed of a stream will be written in the following form [8]

$$v_{av} = \frac{1}{n} \sqrt{d^2\sqrt{l}}$$

All these higher indicated parameters and depending on them formulas are well-known and counted. At the overflow of the flat rivers an important parameter there are speed and volume of the water spilling from the river. Determining speed of water of $v_{cp}$ in the riverbed of the river on the above – mentioned of Shezi’s formula, it is possible to calculate speed of spilling water by a formula:

$$v = v_{av} + \sqrt{2gh\sin \alpha}$$

(6)

where $g$ – acceleration of the free fall, $h$ – difference between spilling water level and depth adherent pool and the $\alpha$ – corner of flow under which water spills.

Using a flood place (as a rule places of ice jams) as an initial point, and knowing the section and speed of the river higher and lower flood points (initial point), it is possible to calculate the volume of water flowing from the river of unit of time $\Delta t$ in a flood point:

$$Q(\Delta t) = (F_{higher} \cdot v_{av,1}) - (F_{lower} \cdot v_{av,2}),$$

(7)

where $F$— area of living section, and $v_{av,1}, v_{av,2}$ — an average speeds of the rivers higher and lower a flood point, respectively. So, the difference between water flows higher and below a point of flood gives the necessary volume of water by a formula (7) flooding the coastal territory of the river for a unit of time.

The volume of following water at flood of the rivers is closely related to such concept as a consumption of water. Water volume (in cubic meters), proceeding through the area of live section in unit of time is called as a consumption of water $Q$: $Q=F \cdot v_{av}$, where $F$ – the area of living section and $v_{av}$ – average speed of flow. Therefore, for definition of a consumption of water it is necessary to determine the area of live section and average speed of a flow. The area of cross section of the stream, limited by the riverbed below, and by a water surface above and located perpendicularly to the direction of flow is called as the area of live section.

Besides, the speed of a watercourse is defined by skilled methods. It can be done in various ways:

1) by superficial floats;
2) through hydrometric poles or milestones;
3) through deep floats;
4) by hydrometric rotators.
Operating with such above-mentioned sizes as river speed \( v_a \), by the speed of flowing water from the river counted by a formula (6), flooding the contiguous territory and the volume of followed water on the basis of a method of consecutive pools it is possible to determine dynamics of movement of water and to construct the map of flooding coastal territories of the rivers depending on time with the subsequent realization on GIS-technologies.

Thus, it is possible to claim that the offered technique of modeling of an emergency on water objects allows to predict the map of flooding of the district depending on time and the speed of replenishment of riverbeds of the rivers.

**Conclusion**

Methods which are widespread at the present time, based on very difficult multiple-factor nonlinear differential equations, even numerical decision which demands powerful hardware for modeling of very small water flows. The method offered in this article allows to hide nonlinearity in natural interrelation between pools and to model the movement of big water flows, without demanding powerful hardware. Thus, this approach allows to model zones of flooding and the water flow direction at various emergency situations on water objects of Kazakhstan.

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**REFERENCES**


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