ON THE ISSUES OF FRACTAL RADIO ELECTRONICS: Part 2. DISTRIBUTION AND SCATTERING OF RADIO WAVES, RADIO HEAT EFFECTS, NEW MODELS, LARGE FRACTAL SYSTEMS

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The paper presents fractal approaches to solving problems of radio electronics at all stages of radiation and reception of radio waves with the subsequent processing of incoming information. In the second part of the article, fractal effects arising from the propagation and scattering of waves in randomly inhomogeneous media are considered. The radar equation for sounding a fractal object is detailed. Models based on fractals and strange attractors for radio wave scattering by plant cover are proposed.

Keywords: radio physics, radio thermal radiation of the atmosphere, radio wave scattering, fractal, scaling, strange attractors, large fractal systems.

Introduction

In this part of the article the author considered fractal effects arising from the propagation and scattering of waves in randomly inhomogeneous media; detailed the radar equation for sounding a fractal object; proposed models based on fractals and strange attractors for radio wave scattering by plant cover, i.e. issues that relate to the whole radio electronics. The study is conducted within the framework of the research area “Fractal Radiophysics and Fractal Radioelectronics: Designing Fractal Radio Systems”, proposed and developed by the author based on the theory of fractals and deterministic chaos in the IRE of the RAS since the late 70s of the XX century [1-4].

1. Fractal effects during wave propagation in the troposphere

When waves propagate in the atmosphere, it is necessary to take into account turbulence and its multi-scale character. In the modern theory of turbulence, it is assumed that the vortex layers coagulate into complex fractal structures [5]. The author, together with the Almaz Central Design Bureau, previously carried out multiscale experimental work on creating a data bank on the spatial-temporal characteristics of millimeter-wave scattering for the subsequent analysis and synthesis of radar image textures using simple and complex phase-manipulated signals of very large database ≥10^6; the contribution of hydrometeor space-time distribution to image characteristics. A series of experiments was carried out on the express analysis of fractal fluctuations of ultra-wideband and simple signals of MMW and CMW in the turbulent troposphere at distance range gating. The average wind speed during field experiments was 3 ± 0.5 m/s. The processing showed that in summertime (air temperature of 20°-25°) on the surface route 150 m long at a height of 10 m and a radiation wavelength of 8.6 mm for amplitude fluctuations, the fractal dimension \( D \approx 1.63 \). In this case, the Hurst parameter is \( H \approx 0.37 \). In the case of radar sounding, the fractal dimension increased to \( D \approx 1.72 \ (H \approx 0.28) \). In case of drizzling rain, the fractal dimension of amplitude fluctuations decreased to values of the order of \( D \approx 1.59 \ (H \approx 0.41) \). The magnitude of the standard deviation is <0.02. In experiments, processes with \( D = 1.5 \) were never detected. Thus, in the course of field experiments, only anti-persistent processes were observed.
2. Fractal properties of the troposphere radiothermal radiation

In the experiments, the researchers measured mainly the vertical absorption at a wavelength of 8.2 mm. A radiometer with a sensitivity of 0.5 K, made according to the modulation scheme with a super heterodyne receiver at the input, provided measurement of the noise signal in the frequency band $\Delta f = 400$ MHz along the principal and image channels at an intermediate frequency of 250 MHz. The modulation of a signal for a frequency of 1000 Hz was carried out using a ferrite switch. A horn with a directional pattern width of $9^0 \times 1^0$ was used as an antenna. Phase portraits for each series were reestablished on the basis of the data obtained and the autocorrelation functions of the studied series were constructed. The analysis of the statistical characteristics of radiothermal radiation was carried out using the Pearson diagram [3, 22]. Further, the fractal dimension and the Hurst index were measured. The numerical estimates of the correlation dimension $D_c$ and the Hurst index $H$ are summarized in the table.

Table 1. Fractal properties of radiothermal radiation in the MMW range

<table>
<thead>
<tr>
<th>Series number</th>
<th>Correlation dimension $D_c$</th>
<th>Hurst index $H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - Clean troposphere</td>
<td>1.7746</td>
<td>0.2254</td>
</tr>
<tr>
<td>2 - Cumulonimbus</td>
<td>1.6509</td>
<td>0.3491</td>
</tr>
<tr>
<td>3 - Stratocumulus</td>
<td>1.7172</td>
<td>0.2828</td>
</tr>
<tr>
<td>4 - Series</td>
<td>1.5190</td>
<td>0.4810</td>
</tr>
<tr>
<td>5 - Series</td>
<td>1.4943</td>
<td>0.5057</td>
</tr>
</tbody>
</table>

The Hurst index, depending on its value relative to the value $H = 1/2$, characterizes either persistence ($1/2 < H < 1$) or antipersistence ($0 < H < 1/2$) of the current sample. In the first case, when $1/2 < H < 1$ we observe a process that preserves the tendency to increase or decrease the instantaneous amplitudes in the sample, i.e. process with memory. In the second case, when $0 < H < 1/2$, increase in the amplitudes of the signal envelope in the "past" means decreasing in the "future", and vice versa, i.e. a more changeable process, often is referred to as a “mean reversion.” The measured value of the Hurst index $H$ (see the table) indicates a high antipersistence of the process of radiothermal radiation for all series of measurements.

3. Wave scattering on a stochastic fractal surface

In solving a number of relevant physical and practical problems, one has to deal with the processes of wave scattering on a statistically uneven surface. The need to introduce fractal models are due to the following reasons [1, 3, 6, 7]:

1. Numerous experiments [1, 3] have established that the majority of surfaces can be classified as fractal ones in certain space-time scales.

2. The fractal surface comprises irregularities of all scales relative to the length of the scattered wave.

3. Fractal models of wave scattering are natural generalization of the two-scale model used in remote sounding and radiolocation.

4. Taking into account fractality significantly approximates the theoretical and experimental angular dependences of the specific effective scattering areas (ESA) $\sigma_e$ [1]. This fact is often interpreted as mainly instrumental error, which is incorrect.
5. The possibility of using non-differentiable functions to describe irregularities; it makes for correct introduction of exponential correlation coefficients.

Next, consider the issues of the theory of wave scattering on a stochastic fractal surface as applied to the problems of the formation of radar images (RI) [6, 7]. In the general case, the radar image can be interpreted as a map (matrix) of ESA $\sigma$, or a signature (portrait) of the object being sounded in the case of a high angular resolution. All the results presented below are of priority in Russia and in the world [1, 3, 6, 7].

The mathematical statement of the problem and the initial relations are given in [1, 6, 7]. The fractal wave front, being non-differentiable, does not have a normal. Thus, the concepts “ray tracing” and “geometrical optics effects” are excluded. In this case, the “topothesy” of a fractal chaotic surface is introduced [1]. The scattering surface is modeled by a range-limited continuous fractal function of irregularities $f(x)$, which is a modified Weierstrass function $W(t)$. Weierstrass’s two-dimensional range-limited function $W(x,y)$ is written as:

$$W(x, y) = c_0 \sum_{n=0}^{N-1} q^{(D-3)n} \sum_{m=1}^{M} \sin \left\{ Kq \left[ x \cos \left( \frac{2\pi n}{M} \right) + y \sin \left( \frac{2\pi n}{M} \right) \right] + \varphi_{nm} \right\},$$

(1)

where $c_0$ is a constant providing a single normalization; $q > 1$ is the parameter of spatial-frequency scaling; $D$ is the fractal dimension ($2 < D < 3$); $\varphi_{nm}$ is an arbitrary phase, uniformly distributed in the interval $[-\pi, \pi]$.

Figure 1 shows the calculated implementations of the Weierstrass function $W(x,y)$ for different scales. The implementation with the fractal dimension $D=2.5$ corresponds to the classical two-dimensional Brownian process (Figure 1 b) [1, 3, 6, 7].

The function $W(x,y)$ is anisotropic in two directions, if $M$ and $N$ are not very large. The function $W(x,y)$ has derivatives and at the same time it is self-similar. Such parameters as the averaged correlation interval $\tilde{\rho}$, mean square deviation $\sigma$ and the spatial autocorrelation coefficient $R(\tau)$ are traditionally used to numerically describe a rough surface.

Figure 2 shows the dependences $\tilde{\rho}$ on the scale factor $q$ and the fractal dimension of the surface $D$, respectively [6, 7]. Spatial scattering indicatrixes of fractal surfaces in the microwave spectrum are currently studied completely insufficiently.

An extensive bank / catalog of typical types of more than 70 fractal scattering surfaces based on Weierstrass functions, as well as three-dimensional scattering indicatrixes and their sections calculated for $\lambda = 2.2 \text{ mm}$, $\lambda = 8.6 \text{ mm}$ and $\lambda = 3.0 \text{ cm}$ wavelengths for different values of dimension $D$ and changing scattering geometry were investigated and presented in [6, 7] for the first time ever. The field of dispersion $\psi_p$ from the final site $S$ is described by the following expression:

$$\psi_p(r) = -\frac{iL_\lambda L_\omega \exp(ikr)}{\pi r} F(\theta_1, \theta_2, \theta_3) \times \sum_{u_0=-\infty}^{+\infty} \sum_{u_1,0=-\infty}^{+\infty} \sum_{u_2,0=-\infty}^{+\infty} \sum_{u_2,N-1=-\infty}^{+\infty} \sum_{u_M,N-1=-\infty}^{+\infty} \times$$

$$\prod_{n=0}^{N-1} \prod_{m=1}^{M} J_{nm} (c_j q^{(D-3)n}) \exp(i \sum_{n=0}^{N-1} \sum_{m=1}^{M} u_{nm} \phi_{nm}) \times \text{sinc}(\varphi_c L_c) \text{sinc}(\varphi_c L_\omega) + \psi_c,$$

(2)

where all designations are presented in [6, 7].
Fig. 1. Examples of Weierstrass function for: a) $N = 2, M = 3, D = 2.01, q = 1.01$; b) $N = 5, M = 5, D = 2.5, q = 3$; c) $N = 10, M = 10, D = 2.99, q = 7$ (c).

Fig. 2. The averaged correlation interval $\rho$ as a function of $D$ for: a) $q = 1.01; q = 1.5; q = 1.8; q = 2; q = 2.5; q = 2.7$ and b) as a function of $q$ for $D = 2.01; D = 2.2; D = 2.3; D = 2.5; D = 2.67; D = 2.8; D = 2.99$.

4. Fractals and strange attractors in models of radio wave scattering by plant cover

Processing the enveloping reflected radar signals at a wavelength of 2.2 mm for the first time made it possible to determine the characteristics of a strange attractor controlling millimeter-wave scattering in the phase space [8], see Figure 3a. Radar data were obtained personally by the author at the test site in field experiments as early as in 1979–1980. When reconstructing this attractor from ordered measurements of one variable, it is necessary to construct an embedding space of dimension $m = 2N_0 + 1$ in order to describe all possible topological features of the attractor.
The value $N_0 \geq \text{int}[D] + 1$ determines the number of differential equations of the 1st order necessary to describe the physical behavior of the dynamical system under study. Here $\text{int}[D]$ is the operation of extracting the integer part of $D$, and $D$ is the true fractal dimension of the strange attractor. The correlation integral $C(r)$ can also be used as a means of separating deterministic chaos and external white noise. For Gaussian noise (Figure 3b) there is no tendency to saturation. Therefore, the attractor of infinite dimension corresponds to it. This distinction is widely used in processing time realizations of unknown origin.

The following values were obtained (Figure 3a): $D = 1 + 1.84 \approx 2.8$; the embedding dimension $m=7$; Lyapunov index $\lambda_1 \geq 0.6$ bits/c; the prediction time $\tau_{max} \approx 1.7$ s at correlation time of the reflected signal intensity $\tau \approx 210$ ms and a wind speed of 3 m/s Therefore, if current conditions are measured with an accuracy up to 1 bit, then all the predictive power in time will be lost in about 1.7 s. At the same time, the prediction interval $\tau_{max}$ of the intensity of the radar signal exceeds the classical correlation time by about 8 times.

The data obtained together with the family of fractal distributions underlie a new dynamic model of radio signals scattered by plant cover. The proposed dynamic model of the electromagnetic wave scattering by earth covers is fundamentally different from the existing classical models. It has a finite number of degrees of freedom, describes the processes of non-
Gaussian scattering and for the first time injects the prediction interval of the intensity of the received radar signal, as well as its fractal characteristics (signatures).

5. "Distributed intelligence" in the team interaction of UAVs

Unmanned aerial vehicles (UAVs) are currently used to solve a wide range of scientific and practical problems and can become the main element in the formation of a common information field [9-11]. The solution of difficult complex tasks is possible only as a result of group use of UAVs. The team is a network of a certain number of similar or different types of UAVs, united by a common target. In practice, it is possible to implement a significant number of types of UAV team flights. The classification of UAV team flights in the process of accomplishing preset tasks is shown in Figure 4. Fractal UAV groupings and fractal flight paths of UAV are also presented there.

![Fig.4 Classification of UAV team flights](image)

In real conditions, UAVs operate in a non-deterministic, unpredictable environment and under counter operation conditions. Each of the UAV performs a series of actions aimed at solving a common task. A parallel should be drawn with biological research that is trying to answer the question of the emergence of cooperative behavior in the process of evolution or the so-called evolutionary strategy [11]. Intellectual or population algorithms (methods) relate to the class of stochastic search optimization algorithms. Population algorithms refer the class of heuristic algorithms for which convergence to a global solution has not been proven, but it has been established experimentally that they provide a fairly good solution.

To teach the UAV to fly like a flock of birds or a swarm of bees is a relevant problem. Then we can control one apparatus, and the rest will be controlled by this technology. And if the leader dies for some reason, the function of the pack leader automatically proceeds to the next vehicle. And so it will be as long as the last instrument operates. For a complex network of multiple micro- (nano-) UAVs, it conducts global monitoring of the territory and objects located on it. The task can be considered within the framework of the concept of a distributed measuring environment, when each point of a certain dynamic environment is capable of performing sensory, measuring and informational functions.

The fractal-graph approach makes it possible to study the growth of complex networks and provides a method for manipulating such networks at the global level, without using a detailed description. At the same time, it turns out that an excessive number of sensors (UAV) does not guarantee their optimal distribution in/along the non-deterministic environment under study. The introduction of the fractal topology of such networks, taking into account the configuration of the study area, will make for monitoring it with the detection of objects, more accurately and using less means (number of UAVs).
6. Waves in disordered large fractal systems

The issues of the general theory of multiple scattering of electromagnetic waves in fractal discrete randomly inhomogeneous media based on modifications of the classical Foldy-Tverskoy theory are considered in detail in [12]. The integral equation for the coherent field and the second moment of the field for the fractal scattering medium respectively are:

\[
\langle \psi^a \rangle = \phi^a + \int u^a_1 \phi^1 w(r) dr + \int \int u^a_i u^i_1 \phi^1 w(r) w(r') dr dr' + \int \int \int u^a_i u^i_1 u^1_2 \phi^2 w(r) w(r') w(r'') dr dr' dr'' + \ldots,
\]

\[
\langle \psi^a \psi^b \rangle = \langle \psi^a \rangle \langle \psi^b \rangle + \int v^a_s v^b_s \langle \psi^s \rangle \langle \psi^s \rangle + \int v^a_s v^b_s v^s_1 v^s_2 \langle \psi^s \rangle \langle \psi^s \rangle + \int \int \int \int \int v^a_s v^b_s v^s_1 v^s_2 v^s_3 v^s_4 \langle \psi^s \rangle \langle \psi^s \rangle \langle \psi^s \rangle \langle \psi^s \rangle \langle \psi^s \rangle \langle \psi^s \rangle + \ldots,
\]

where \( v^a_s = u^a_2 + \int u^a_3 v^3_s w(r) dr \), \( w(r) = \left( \frac{r}{r_0} \right)^{E-D} \), other notations are given in [12].

The data of [12] aim to summarize the study of how the problem of multiple scattering of waves on an ensemble of particles is solved and to what extent it is possible to obtain a solution to this problem for the modern theory of multiple scattering of waves in fractal discrete randomly inhomogeneous media when solving problems of modern radiolocation. On the basis of formula (3), the basic concepts of a fractal medium are introduced and the formulation of the mathematics of multiple scattering of electromagnetic waves in a fractal medium is given simultaneously with the physics of the scattering process. The constructed modification of the multiple scattering theory made it possible to include in the consideration the values of the fractal dimension \( D \) and the fractal signature \( D(r,t) \) of an unordered large fractal system.

7. The radar equation for a fractal target

Calculate the value of the backscattered signal from the fractal medium [12] using the classical radar equation. The received signal power \( P_s \) is determined by the radar equation. Here we have two cases:

a) for the far-field region and the two dimensional fractal target (Euclidean dimension \( E=2 \)); then

\[
P_s \propto \frac{1}{r^{4-D}}.
\]  

b) for the far-field region and the three dimensional fractal target (Euclidean dimension \( E=3 \)); then

\[
P_s \propto \frac{1}{r^{5-D}}.
\]

where \( r \) is the distance to the target.

The results (4) and (5) show that the reflected radar signal can be used to estimate the fractal dimension \( D \) of the sounded fractal medium or fractal target (such as a

Similarly, on the base of (3)-(5), it is possible to obtain a solution for anisotropic disordered large fractal systems: fractal cascades put one into another, graphs from fractal chains, percolation systems, nanosystems, space debris, clusters of unmanned aerial vehicles or small space vehicles (SSV), of mini- and microclasses as well, dynamic synthesized space antenna groups (cluster apertures), low-observable high-altitude pseudo-satellites (HAPS), space-distributed space systems (clusters) from SSVs for solving problems of emergency situations monitoring, etc.
Conclusion

Fractal effects arising from propagation and scattering of waves in randomly inhomogeneous media are considered. Radar equation for sounding a fractal object is detailed. Models based on fractals and strange attractors for radio wave scattering by plant cover are proposed.

This research continues the cycle of studies in order to establish the application of the fractal theory, physical scaling and fractional operators in issues of radio physics and radio electronics, started by the author for the first time in the USSR at the IRE of the USSR Academy of Sciences in the late 1970s. Careful bibliographic studies prove the author’s complete and absolute priority in the world in all “fractal” areas in radio physics and radio electronics. This article, together with [13], provides basic information on the current state in the application of the fractal theory and fractional calculus for innovative technologies.

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REFERENCES


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